



AP[®] Physics C: Mechanics

2016 Free-Response Questions

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ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23} \text{ mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11} (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31 \text{ J}/(\text{mol}\cdot\text{K})$	Acceleration due to gravity at Earth's surface, $g = 9.8 \text{ m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23} \text{ J/K}$	
1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^3 \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant, $k = 1/(4\pi\epsilon_0) = 9.0 \times 10^9 (\text{N}\cdot\text{m}^2)/\text{C}^2$	
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant, $k' = \mu_0/(4\pi) = 1 \times 10^{-7} (\text{T}\cdot\text{m})/\text{A}$	
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^5 \text{ N/m}^2 = 1.0 \times 10^5 \text{ Pa}$

UNIT SYMBOLS	meter, m	mole, mol	watt, W	farad, F
	kilogram, kg	hertz, Hz	coulomb, C	tesla, T
	second, s	newton, N	volt, V	degree Celsius, °C
	ampere, A	pascal, Pa	ohm, Ω	electron volt, eV
	kelvin, K	joule, J	henry, H	

PREFIXES		
Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin \theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan \theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

- I. The frame of reference of any problem is inertial unless otherwise stated.
- II. The direction of current is the direction in which positive charges would drift.
- III. The electric potential is zero at an infinite distance from an isolated point charge.
- IV. All batteries and meters are ideal unless otherwise stated.
- V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{J} = \int \vec{F} dt = \Delta \vec{p}$$

$$\vec{p} = m\vec{v}$$

$$|\vec{F}_f| \leq \mu |\vec{F}_N|$$

$$\Delta E = W = \int \vec{F} \cdot d\vec{r}$$

$$K = \frac{1}{2} mv^2$$

$$P = \frac{dE}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

$$\Delta U_g = mg\Delta h$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$I = \int r^2 dm = \sum mr^2$$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$v = r\omega$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$K = \frac{1}{2} I\omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

a = acceleration
 E = energy
 F = force
 f = frequency
 h = height
 I = rotational inertia
 J = impulse
 K = kinetic energy
 k = spring constant
 ℓ = length
 L = angular momentum
 m = mass
 P = power
 p = momentum
 r = radius or distance
 T = period
 t = time
 U = potential energy
 v = velocity or speed
 W = work done on a system
 x = position
 μ = coefficient of friction
 θ = angle
 τ = torque
 ω = angular speed
 α = angular acceleration
 ϕ = phase angle

$$\vec{F}_s = -k\Delta \vec{x}$$

$$U_s = \frac{1}{2} k(\Delta x)^2$$

$$x = x_{\max} \cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

$$T_p = 2\pi \sqrt{\frac{\ell}{g}}$$

$$|\vec{F}_G| = \frac{Gm_1 m_2}{r^2}$$

$$U_G = -\frac{Gm_1 m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$|\vec{F}_E| = \frac{1}{4\pi\epsilon_0} \left| \frac{q_1 q_2}{r^2} \right|$$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$E_x = -\frac{dV}{dx}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

$$U_E = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$\Delta V = \frac{Q}{C}$$

$$C = \frac{\kappa \epsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

$$R = \frac{\rho \ell}{A}$$

$$\vec{E} = \rho \vec{J}$$

$$I = Nev_d A$$

$$I = \frac{\Delta V}{R}$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = I\Delta V$$

A = area
 B = magnetic field
 C = capacitance
 d = distance
 E = electric field
 $\mathcal{E}$ = emf
 F = force
 I = current
 J = current density
 L = inductance
 ℓ = length
 n = number of loops of wire per unit length
 N = number of charge carriers per unit volume
 P = power
 Q = charge
 q = point charge
 R = resistance
 r = radius or distance
 t = time
 U = potential or stored energy
 V = electric potential
 v = velocity or speed
 ρ = resistivity
 Φ = flux
 κ = dielectric constant

$$\vec{F}_M = q\vec{v} \times \vec{B}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

$$\vec{F} = \int I d\vec{\ell} \times \vec{B}$$

$$B_s = \mu_0 nI$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$U_L = \frac{1}{2} LI^2$$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$$A = bh$$

Triangle

$$A = \frac{1}{2}bh$$

Circle

$$A = \pi r^2$$

$$C = 2\pi r$$

$$s = r\theta$$

Rectangular Solid

$$V = \ell wh$$

Cylinder

$$V = \pi r^2 \ell$$

$$S = 2\pi r \ell + 2\pi r^2$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

Right Triangle

$$a^2 + b^2 = c^2$$

$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{a}{b}$$

A = area

C = circumference

V = volume

S = surface area

b = base

h = height

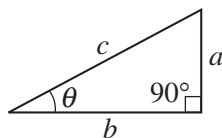
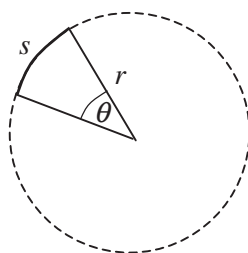
ℓ = length

w = width

r = radius

s = arc length

θ = angle



CALCULUS

$$\frac{df}{dx} = \frac{df}{du} \frac{du}{dx}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a \cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a \sin(ax)$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax)$$

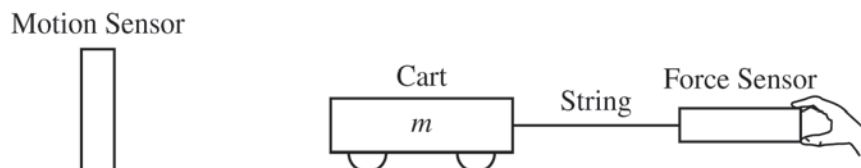
VECTOR PRODUCTS

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$|\vec{A} \times \vec{B}| = AB \sin \theta$$

2016 AP[®] PHYSICS C: MECHANICS FREE-RESPONSE QUESTIONS**PHYSICS C: MECHANICS****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Mech.1.

A cart of mass m is pulled along a level dynamics track as shown above. A force sensor is attached to the cart with a string and used to measure the horizontal force exerted on the cart to the right. A motion sensor is used to measure the acceleration of the cart with the positive direction toward the right. Friction is not negligible.

- (a) On the dot below, which represents the cart, draw and label the forces (not components) that act on the cart. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



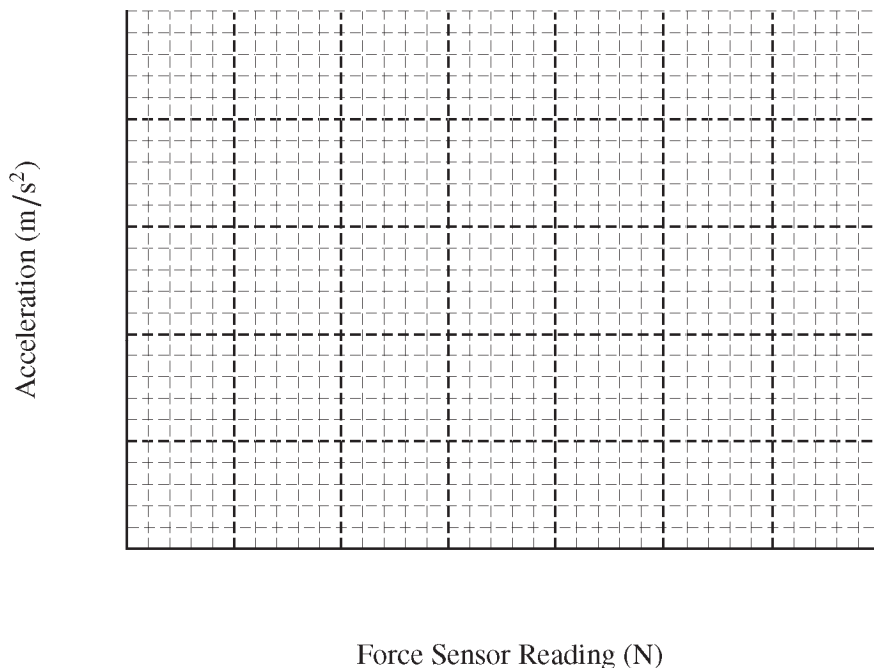
A student pulls the force sensor with a constant force, and the cart accelerates. This is repeated for several trials, with a different constant force used for each trial. The data are recorded in the table below.

Trial	1	2	3	4	5
Force sensor reading (N)	0.32	0.38	0.44	0.50	0.60
Acceleration (m/s^2)	0.12	0.22	0.33	0.50	0.70

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(b)

- i. On the grid below, plot data points for the acceleration of the cart as a function of the force sensor reading. Clearly scale all axes. Draw a straight line that best represents the data.



- ii. Using the straight line from the graph, calculate the mass of the cart.
- iii. Using the straight line from the graph, determine the magnitude of the force of friction.

The above experiment is repeated by using a constant force sensor reading of 0.45 N. The cart starts from rest at time $t = 0$ s and is pulled for a time of 2.0 s along the dynamics track.

(c)

- i. Determine the acceleration of the cart.
- ii. The string breaks at time $t = 2.0$ s. Calculate the time it takes for the cart to stop after the string breaks.

The experiment and analysis in parts (a) and (b) are repeated with a cart that has the same mass but a greater force of friction.

(d)

- i. Will the slope of your new line be greater than, less than, or equal to the slope of your line in part (b)i?
 _____ Greater than _____ Less than _____ Equal to
- ii. Will the horizontal intercept of your new line be greater than, less than, or equal to the horizontal intercept of your line in part (b)i?
 _____ Greater than _____ Less than _____ Equal to

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Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

- (a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Block of Mass $2M$

Block of Mass $3M$



The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D , as shown above. Express your answers to parts (b), (c), and (d) in terms of M , B , v_0 , and physical constants, as appropriate.

- (b) Derive an expression for the speed of the blocks immediately after the collision.
 (c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.
 (d) Derive an expression for the maximum distance D that the spring is compressed.

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(e)

- i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

_____ Left _____ Right _____ The net force on the block of mass $2M$ is zero.

Justify your answer.

- ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

_____ The magnitude of the net force is greater on the block of mass $2M$.
_____ The magnitude of the net force is greater on the block of mass $3M$.
_____ The magnitude of the net force on each block has the same nonzero value.
_____ The magnitude of the net force on each block is zero.

Justify your answer.

- (f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

_____ Yes _____ No

Justify your answer.

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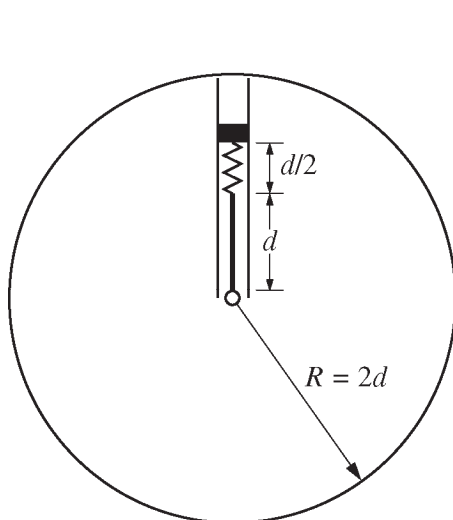


Figure 1

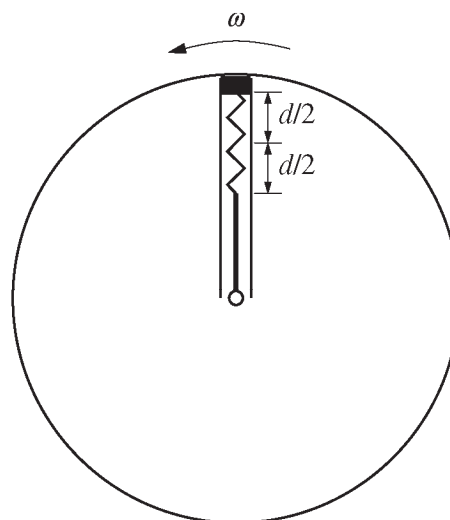


Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω . Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

- Derive an expression for the spring constant of the spring.
- Determine an expression for the rotational inertia of the block around the axis of the platform.
 - Derive an expression for the rotational inertia of the entire system about the axis of the platform.
- Determine an expression for the angular momentum of the entire system about the axis of the platform.

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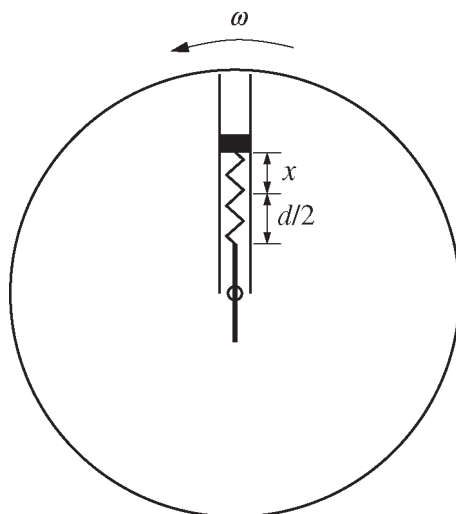


Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

- (d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

- (e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Justify your answer.

- (f) In order to keep the system rotating with constant angular speed ω , is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Justify your answer.

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- (g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

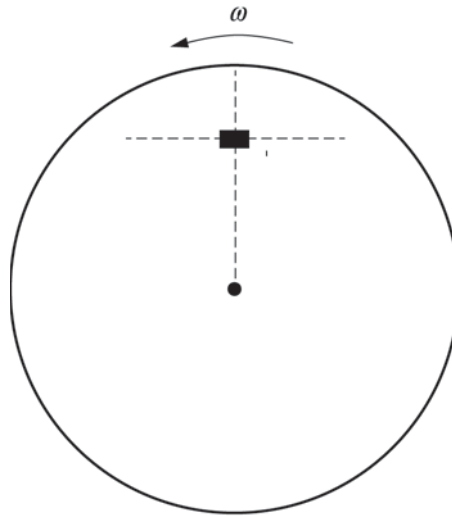


Figure 4

STOP

END OF EXAM